

No-Bang Quantum State of the Cosmos ^{*}

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(2008 March 7)

Abstract

A quantum state of the entire cosmos (universe or multiverse) is proposed which is the equal mixture of the Giddings-Marolf states that are asymptotically single de Sitter spacetimes in both past and future and are regular on the throat or neck of minimal three-volume. That is, states are excluded that have a big bang or big crunch or which split into multiple asymptotic de Sitter spacetimes. (For simplicity, transitions between different values of the cosmological constant are assumed not to occur, though different positive values are allowed.) The entropy of this mixed state appears to be of the order of the three-fourths power of the Bekenstein-Hawking $A/4$ entropy of de Sitter spacetime. Most of the component pure states do not have rapid inflation, but when an inflaton is present and the states are weighted by the volume at the end of inflation, a much smaller number of states may dominate and give a large amount of inflation and hence may agree with observations.

^{*}Alberta-Thy-08-07, arXiv:0707.2081

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Introduction

Our observations strongly suggest that our observed portion (or subuniverse [1] or bubble universe [2, 3] or pocket universe [4]) of the entire universe (or multiverse [5, 6, 7, 8, 9, 10, 11, 12] or metauniverse [13] or omnium [14] or megaverse [15] or holocosm [16], though I am henceforth abandoning this neologism after my wife warned me recently that it might potentially be interpreted as offensive because of its similarity to holocaust, which was certainly not my intent, and I apologize to anyone whom I might have offended) is much more special than is implied purely by the known dynamical laws. For example, it is seen to be enormously larger than the Planck scale, with small large-scale curvature, and with approximate homogeneity and isotropy of the matter distribution on the largest scales that we can see today. It especially seems to have had extraordinarily high order in the early universe to enable its coarse-grained entropy to increase and to give us the observed second law of thermodynamics [17, 18, 19].

Leading proposals for special quantum states of the universe have been the Hartle-Hawking ‘no-boundary’ proposal [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30] and the ‘tunneling’ proposals of Vilenkin, Linde, and others [31, 32, 33, 34, 35, 36]. In simplified toy models with a suitable inflaton, both of these classes of models have seemed to lead to the special observed features of our universe noted above.

However, Leonard Susskind [37] (cf. [38, 39, 40]) has made the argument, which I have elaborated [41], that in the no-boundary proposal the cosmological constant or quintessence or dark energy that is the source of the present observations of the cosmic acceleration [42, 43, 44, 45, 46, 47, 48] would give a large Euclidean 4-hemisphere as an extremum of the Hartle-Hawking path integral that would apparently swamp the extremum from rapid early inflation. Therefore, to very high probability, the present universe should be very nearly empty de Sitter spacetime, which is certainly not what we observe.

The tunneling proposals have also been criticized for various problems [49, 50, 51, 52, 53, 54]. For example, the main difference from the Hartle-Hawking no-boundary proposal seems to be the sign of the Euclidean action [32, 31]. It then seems problematic to take the opposite sign for inhomogeneous and/or anisotropic perturbations without leading to some instabilities, and it is not clear how to give a sharp distinction between the modes that are supposed to have the reversed sign of the action and the modes that are supposed to retain the usual sign of the action. Vilenkin has emphasized [31, 36] that the instabilities do not seem to apply to his particular tunneling proposal, which does not just reverse the sign of the Euclidean action. However, Vilenkin admits [36] that “both wavefunctions are far from being rigorous mathematical objects with clearly specified calculational procedures. Except in the simplest models, the actual calculations of ψ_T and ψ_{HH} involve additional assumptions which appear reasonable, but are not really well justified.”

Therefore, at least unless and until any of these proposals can be made rigorous and can be shown conclusively to avoid the problems attributed to them, it is worth searching for and examining other possibilities for the quantum state of the universe or multiverse. In this paper a schematic proposal for another state is made for uni-

verses with positive cosmological constant, based upon the global picture of ‘eternal de Sitter space’ given recently by Giddings and Marolf [55]. They assert, “Eternal de Sitter physics is described by a finite dimensional Hilbert space in which *each* state is precisely invariant under the full de Sitter group.” Here I make a further restriction of the states and then propose that the quantum state of the universe is the uniform mixture of all of the resulting finite set of pure states. Since none of these pure states have the entire universe beginning at a big bang or ending at a big crunch, I call the resulting mixed state the ‘no-bang state.’ This state appears to be a quantitative improvement over the no-boundary proposal, but unfortunately it still seems to suffer from a similar qualitative problem.

For simplicity, I shall assume that asymptotically de Sitter spacetimes are absolutely stable, with no transitions between different values of the cosmological constant, though if different positive values of the cosmological constant are allowed, the no-bang state includes all of them. I shall neglect zero and negative values of the cosmological constant and therefore exclude states in which they occur (though I am not opposed to a possible extension of the no-bang state that includes components with zero and negative cosmological constant, if one can make some definite proposal for them).

1 Giddings-Marolf eternal de Sitter states

Giddings and Marolf [55] show how one may use group averaging to convert a dS-noninvariant ‘seed’ state of matter to a dS-invariant state, as all physical states must be in quantum gravity. (Actually, all physical states must be invariant under the full diffeomorphism group, but the de Sitter group is a crucial subgroup of this when there is a positive cosmological constant, as is assumed here.) Giddings and Marolf then consider seed states with weak back-reaction and focus on quasiclassical seed states in each of which the stress-energy tensor has fairly definite values at each point in the background de Sitter space. (After the group averaging, the resulting dS-invariant physical state will give an identical superposition of stress-energy tensor values at all points of the spacetime and in all local Lorentz frames, so it is only the seed states that can be inhomogeneous quasiclassical states in the sense of having fairly definite field values that vary over the spacetime.)

Giddings and Marolf [55] choose a particular foliation of de Sitter space (say 4-dimensional, as I shall assume for simplicity here, though the arguments may be modified simply for any higher dimension) by hypersurfaces that are 3-spheres of symmetry. The sequence of these hypersurfaces shrinks down from asymptotically infinite size at past timelike infinity to a ‘neck’ of minimal size and then re-expands back to asymptotically infinite size at future timelike infinity. Since the background de Sitter space is globally hyperbolic, the seed state over the entire space will be causally determined by its restriction to the neck (at least when one ignores gravity, and presumably even when gravity is included, as shall be assumed here). This restriction to the minimal 3-sphere neck can be considered to be the ‘initial’ data for the seed state, though of course it is temporally before just the part of the

spacetime to the future of the neck. (For the part of the spacetime to the past of the neck, this ‘initial’ state might instead be considered a ‘final’ state, though it is not the final hypersurface on which the seed state may be evaluated either.)

Giddings and Marolf [55] consider seed states which have small gravitational back-reaction at the neck. By calculating the entropy of thermal massless radiation of energy density less than that of the cosmological constant Λ , they estimate that the logarithm of the number of such states is, in 4 dimensions, roughly $(R/\ell_p)^{3/2}$ (a quantity which had also been obtained earlier by Banks, Fischler, and Paban [56]), where R is the radius of the neck (given by $b \equiv \sqrt{3/\Lambda}$ when there is negligible gravitational back-reaction), and ℓ_p is the Planck length. This logarithm is much less than the Bekenstein-Hawking entropy of de Sitter space, which is $\pi(R/\ell_p)^2$, one quarter the area of the cosmological horizon in Planck units ($\hbar = c = G = k_{\text{Boltzmann}} = 1$). The much larger latter entropy suggests [55] that there are far more non-perturbative gravitational states, perhaps black holes of very large size, comparable to R .

Since states with black holes of very large size do not seem to fit our observations of the universe, I would like to exclude them from the mixed state I am proposing. However, I would prefer not simply to restrict to states of small gravitational back-reaction, first because it is rather arbitrary how to define “small,” and second because I would not want to exclude stellar-mass black holes in the present universe, which certainly do not have small gravitational back-reaction.

2 No-bang seed states

My proposal is to consider seed states for which one has a regular neck, and for which the seed state evolves to a single asymptotically de Sitter space in both the infinite past and the infinite future, without the entire universe having either a big bang in the past or a big crunch in the future, and without the universe splitting up into more than one asymptotically de Sitter space in either the infinite past and the infinite future as it would do for necks with large black holes. However, no restriction is proposed that gravity need be weak on the neck, or that no black holes occur either to the past or the future of this neck. (I shall assume that all holes do decay away in both the asymptotic past and future, say by Hawking radiation in the future and its time reverse in the past.)

Here the neck is defined as a complete closed 3-dimensional locally-minimal hypersurface (a complete closed hypersurface with zero mean extrinsic and with smaller 3-volume than any other nearby complete closed hypersurface with constant mean extrinsic curvature), the hypersurface of globally minimal 3-volume if more than one such locally-minimal hypersurfaces occur. Since my discussion will be almost entirely at the semiclassical level, I shall assume that each seed state gives a spacetime geometry that is sufficiently quasiclassical, at least in the neighborhood of the neck, that one can get a good approximate description of the spacetime geometry in that region. (Elsewhere in the spacetime, such as where quantum uncertainties in gravitational collapse may occur, as in the formation and evaporation of black holes, one

need not be able to describe the seed state as giving a single approximately classical geometry.)

3 Homogeneous, isotropic necks

First I shall consider the case in which the neck is approximately a homogeneous, isotropic 3-sphere of radius R , which Giddings and Marolf [55] used to estimate the entropy of seed states of weak gravitational back-reaction. For now I shall follow them in assuming that the perturbations from homogeneity and isotropy are small, but I shall allow arbitrarily large (though nearly homogeneous and isotropic) matter stress tensors and arbitrarily large back-reaction upon the size of the neck and the subsequent evolution of the spacetime.

I shall use quantum units in which $\hbar = c = k_{\text{Boltzmann}} = 1$, but since there are different conventions as to whether the Planck length ℓ_p is $G^{1/2}$ (my usual convention) or $(8\pi G)^{1/2}$, I shall in this paper explicitly include the factors of Newton's gravitational constant G .

I shall assume that there is a positive cosmological constant $\Lambda = 3/b^2 \ll G^{-1}$ with characteristic length scale much larger than the Planck length, $b \gg G^{1/2}$ (the radius of the resulting de Sitter spacetime if no matter or gravitational waves were present). I shall assume that the matter most relevant near the neck consists of an inflaton field plus thermal radiation. For simplicity, I shall assume that the inflaton scalar field ϕ is a homogeneous free massive scalar field of mass m whose Compton wavelength is much less than b but much greater than the Planck length, $\Lambda \ll m^2 \ll G^{-1}$. Its energy density is $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$, and its isotropic pressure is $P_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$, where an overdot denotes a derivative with respect to the proper time that is measured normally to the homogeneous, isotropic 3-spheres of radius $R(t)$ that foliate the universe near the neck ($t = 0$). I shall assume that the thermal radiation is also homogeneous and isotropic and is dominated by fields whose mass can be neglected, so that its energy density is $\rho_{\text{rad}} = aT^4$ and its isotropic pressure is $P_{\text{rad}} = (1/3)aT^4$, where T is the temperature (which evolves proportional to $1/R$ away from the neck), and a is the radiation constant (e.g., $a = 427\pi^2/120$ for the fields of the Standard Model of particle physics at high temperature [57]).

Thus I assume that near the neck, the spacetime geometry is given by a closed ($k = +1$) FRW model with time-dependent radius $R(t)$. The Einstein constraint equation is

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{4\pi G}{3}(m^2\phi^2 + \dot{\phi}^2 + 2aT^4) + \frac{\Lambda}{3} - \frac{1}{R^2}, \quad (1)$$

and the equation of evolution is

$$\frac{\ddot{R}}{R} = \frac{4\pi G}{3}(m^2\phi^2 - 2\dot{\phi}^2 - 2aT^4) + \frac{\Lambda}{3}. \quad (2)$$

The neck has $\dot{R} = 0$, so at the neck

$$R = \left(\frac{4\pi G}{3}\right)^{-1/2} \left(m^2\phi^2 + \dot{\phi}^2 + 2aT^4 + \frac{\Lambda}{4\pi G}\right)^{-1/2}. \quad (3)$$

For the neck to be a minimal 3-sphere, one needs $\ddot{R} \geq 0$, which puts a limitation on $\dot{\phi}$ and T . This has the implication that the minisuperspace constrained phase space [58] for the FRW model with the cosmological constant and the inflaton without the thermal radiation is $2\pi^2 R^3 d\phi \wedge d\dot{\phi}$, which when using Eq. (1) integrates to a finite total measure over this phase space, under the condition that there be a neck.

When one puts the radiation back in, most of the states will come from the entropy of the radiation. For fixed ϕ , the maximum entropy of the radiation will come from having the maximum value of the temperature T consistent with $\ddot{R} \geq 0$ at $\dot{\phi} = 0$, which gives

$$T = (2a)^{-1/4} \left(m^2\phi^2 + \frac{\Lambda}{4\pi G}\right)^{1/4}, \quad (4)$$

$$R = \left(\frac{8\pi G}{3}\right)^{-1/2} \left(m^2\phi^2 + \frac{\Lambda}{4\pi G}\right)^{-1/2}. \quad (5)$$

Since the 3-volume of the neck is $V = 2\pi^2 R^3$, and since the entropy of the radiation is $S = (4/3)aT^3V$, one finds that the maximum entropy for each value of the inflaton field ϕ at the neck is

$$S = \left(\frac{9\pi^2 a}{512}\right)^{1/4} \left(G^2 m^2 \phi^2 + \frac{G\Lambda}{4\pi}\right)^{-3/4}. \quad (6)$$

For example, for $\phi = 0$, $T = [\Lambda/(8\pi Ga)]^{1/4} = [3/(8\pi Ga)]^{1/4} b^{-1/2}$ and $R = (2\Lambda/3)^{-1/2} = b/\sqrt{2}$ (slightly smaller than the de Sitter radius $b = \sqrt{3/\Lambda}$ at $T = 0$, since the maximal thermal radiation for $\ddot{R} \geq 0$ shrinks the size of the neck), so

$$S = S_{\text{no-bang}} = \left(\frac{9\pi^5 a}{8G^3 \Lambda^3}\right)^{\frac{1}{4}} = [\pi^5 a/(24G^3)]^{1/4} b^{3/2} = [\pi^5 a/(3G^3)]^{1/4} R^{3/2}, \quad (7)$$

agreeing with the order-of-magnitude estimate of Giddings and Marolf [55] for the maximal entropy of states with weak gravitational back-reaction. Here we get the same order-of-magnitude limitation as they did, but now for the entropy of all states with approximate 3-sphere symmetry at the neck, whether or not the homogeneous, isotropic stress-energy tensor of the inflaton plus radiation is a weak perturbation of de Sitter spacetime. (However, see below for how highly anisotropic necks can have arbitrarily large radiation entropy, though they will not evolve to single asymptotically de Sitter regions.)

If we take $\Omega_\Lambda = 0.72 \pm 0.04$ from the third-year WMAP results of [46] and $H_0 = 72 \pm 8$ km/s/Mpc from the Hubble Space Telescope key project [59], and

drop the error uncertainties, we get $G\Lambda = 3\Omega_\Lambda H_0^2 \approx 3.4 \times 10^{-122}$, which would give $b = \sqrt{3/\Lambda} \approx 9.4 \times 10^{60} \sqrt{G}$. This then gives an entropy of the part of the no-bang mixed state corresponding to our value of the cosmological constant of $S_{\text{no-bang}} \approx 4 \times 10^{90}$.

Although this is very roughly the same value as the entropy in the cosmic microwave background photons within the observable universe (by the perhaps anthropically explained coincidence that the photon energy density today is within a few orders of magnitude of the cosmological constant), conceptually this is not the same quantity. On one hand, it represents the von Neumann entropy of the entire spacetime that might be much larger than that of our observed region. On the other hand, the entropy in the cosmic microwave background photons within the observable universe is a coarse-grained entropy and also might be much larger than the fine-grained von Neumann entropy of the same region.

For large values of the inflaton at the neck, $m^2\phi^2 \gg \Lambda/(4\pi G)$, although the maximum temperature rises as $\phi^{1/2}$, the 3-volume of the neck shrinks as ϕ^{-3} , so the radiation entropy, $S = (8\pi^2 a/3)(RT)^3$, shrinks as $\phi^{-3/2}$. Therefore, most of these seed states with a nearly homogeneous, isotropic neck have only low values of the inflaton field and hence negligible amounts of (rapid) inflation from the inflaton (not counting the much slower asymptotic exponential expansion from the cosmological constant Λ itself as inflation). That is, for these ‘eternal de Sitter space’ states, inflation is highly improbable, analogous to the restricted class of minisuperspace states used in [60] but different from the ambiguous probability of inflation for all minisuperspace states [61], where both the inflationary and non-inflationary solutions have infinite measure.

On the other hand, if one looks at this FRW universe not at the neck but at a time after the possible rapid inflation (say at the end of inflation, or at some fixed value of the matter energy density that is lower than m^2 to avoid looking during inflation), then the volume of space will be much larger if rapid inflation has occurred. Therefore, if one wants the expectation value of the volume of space at the end of inflation, one should weight each state by the volume then. A motivation for doing this would be Vilenkin’s Principle of Mediocrity [13] that we are a typical civilization, along with the assumption that the number of civilizations would be proportional to the volume of space at the end of inflation. Other motivations are given in [62, 63, 64, 65, 66, 67, 68, 69, 70, 30], though there is also disagreement [60].

If we start with a value of the free massive inflaton field that gives $\phi^2 \gg G^{-1} \gg \Lambda/(Gm^2)$, then most of the rapid inflation will occur during the slow roll phase in which $H \approx \sqrt{4\pi G/3} m\phi$, and the Klein-Gordon equation for the inflaton, $\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$, will give $\dot{\phi} \approx -m^2\phi/(3H)$ and hence $dt \approx -3Hd\phi/(m^2\phi)$, so the number of e-folds of inflation from some large initial ϕ to the end of inflation at $\phi \sim G^{-1/2}$ will be [65, 71]

$$N = \int H dt \approx 2\pi G\phi^2. \quad (8)$$

Therefore, the volume of space at the end of inflation with a free massive inflaton

of initial value ϕ will be $V \sim (m\phi)^{-3}e^{3N} \sim \exp(6\pi G\phi^2)$, dropping the prefactor in the final expression as having a logarithm much closer to zero than the enormous exponential. Here, to get finite answers, I initially ignore quantum fluctuations of the inflaton field during inflation that might lead to an arbitrarily large volume from eternal inflation [62, 63, 64, 65, 66, 67, 68, 69].

When one uses this slow-roll inflationary volume $V \sim \exp(6\pi G\phi^2)$ along with e^S for the number of states, the resulting expectation value of the volume at the end of inflation would be roughly

$$\langle V \rangle \sim \frac{\int V e^S d\phi}{\int e^S d\phi}. \quad (9)$$

It would be somewhat better to include the minisuperspace constrained phase-space factor [58] $2\pi^2 R^3 d\phi \wedge d\dot{\phi}$, which by itself integrates to a finite value when one includes the restriction $\dot{R} \geq 0$ that limits $\dot{\phi}$ at the neck, as discussed above, making the denominator of the right hand side of Eq. (9) finite, even after integrating over an infinite range of ϕ . However, since V grows so rapidly with ϕ (the value of the instanton field at the neck), qualitatively it is not very important to be so refined.

If one takes the integrals in Eq. (9) to be over an infinite range of ϕ , then the numerator will diverge violently, so that $\langle V \rangle$ will be infinite. To get a finite answer, one might cut off the integral at a value of ϕ corresponding to, say, ϵ times the Planck energy density G^{-2} , where ϵ is some number of order unity, so $(1/2)m^2\phi^2 < \epsilon G^{-2}$. Then the slow-roll inflationary volume will be $V < \exp(12\pi\epsilon G^{-1}m^{-2})$. If one uses the estimate that $m \approx 1.5 \times 10^{-6}G^{-1/2} \approx 7.5 \times 10^{-6}(8\pi G)^{-1/2}$ [65, 71] from the measured fluctuations of the cosmic microwave background, one then gets $V < \exp(1.7 \times 10^{13}\epsilon)$.

Since the entropy S of the thermal radiation at the large value of ϕ at the neck needed to give this maximal slow-roll inflationary value is of the order of unity, the contribution to the integral in the numerator of Eq. (9) near the maximum of the restricted range of ϕ is of very roughly of the order of $\exp(1.7 \times 10^{13}\epsilon)$. On the other hand, the contribution to the integral from small values of the inflaton field ϕ , say $m^2\phi^2 < \Lambda/G$, will have huge radiation entropy values, $S \approx S_{\text{no-bang}} \approx 4 \times 10^{90}$, so this region of the integral will give a contribution that is of very roughly of the order of $\exp(4 \times 10^{90})$, which is far, far larger than the contribution near the maximum value of ϕ (if the maximum inflaton field is indeed limited by energy densities near the Planck value, i.e., for $\epsilon \ll 10^{77}$).

Therefore, even if we weight by the 3-volume V , say at some fiducial energy matter energy density less than $\Lambda/8\pi G$ in order to have this energy density possible even without inflation, the non-inflationary solutions overwhelmingly dominate, when we use the slow-roll inflation value $3N \approx 6\pi G\phi^2$ for the logarithm of the growth factor of the volume of the universe during inflation from a large initial value ϕ .

To avoid this conclusion for the no-bang quantum state, we can appeal to eternal inflation [62, 63, 64, 65, 66, 67, 68, 69], which predicts that quantum fluctuations of the inflaton field will lead to an arbitrarily large amount of inflation and hence $\langle V \rangle = \infty$. Then one would indeed get that the inflationary solutions dominate when one weights by the volume.

In order that eternal inflation occur with high probability [62, 63, 64, 65, 66, 67, 68, 69], one needs the inflaton to start in the quantum diffusion regime, which for a free massive scalar inflaton gives $\phi > \phi_q \sim G^{-3/4} m^{-1/2} \sim 10^3 G^{-1/2}$. Restricting ϕ in this way on the neck then restricts the entropy of the thermal radiation on the neck to be $S \sim (Gm^2)^{-3/8} \sim 10^{9/2}$. Thus we would need a few tens of thousands of bits of information to say which of the pure states in the no-bang mixed state with $\phi > \phi_q$ we are in. However, because eternal inflation would spread this information over an enormous volume, we would be unlikely ever to be able to determine it.

In comparison, the logarithm of the number of possible sequences of the roughly 3 billion DNA base pairs in the human genome is $\ln(4^{3 \times 10^9}) \sim 4 \times 10^9$, around a hundred thousand times greater than the eternal inflation universe entropy calculated above. In this sense, the information needed to specify which of these e^S pure states one is in is hundreds of thousands of times less than the information needed to specify one's genome.

(This estimate does not consider possible compression of the information. Since one would expect that only a very tiny fraction of genomes of 3 billion base pairs would be viable, the compressed information to specify which viable genome one has is likely to be many times smaller than the 6 billion bits just needed to list the sequence. It is surely an open question whether the human genome information could be compressed down to several tens of thousands of bits, comparable to the uncompressed information to specify which of the e^S pure states we are in, which might also be compressible.)

On the other hand, the tens of thousands of bits of information needed to specify which pure state out of e^S is considerably more than the few hundred to few thousand bits of information needed to specify which of the $N_{\text{vac}} \sim 10^{100} - 10^{1000}$ or so string vacua [72, 73, 74, 75] we are near. Thus to say which pure state we are in, as well as what the underlying vacuum is, might involve much fewer than a million bits of information, assuming that the no-bang mixed state is correct and that we focus on the states dominating the volume. Publishing these bits in a short paper would not be difficult if one had them, but it is likely to be difficult to determine them.

Here we have used the anthropic argument for large volume to restrict the number of states enormously from the $e^{S_{\text{no-bang}}} \sim \exp(4 \times 10^{90})$ number of states (for our value of the cosmological constant; even much larger for vacua with lower values) to $e^S \sim \exp(10^{4.5})$ volume-weighted inflationary states if the no-bang mixed state is correct. To specify one out of all the first set of non-anthropic states would require billions of times more information than even the entire number of atoms in the observable part of our universe. Since almost all of these enormously many states do not have (rapid) inflation, the no-bang quantum state is one example in which inflation is extremely improbable when one considers all possibilities but is highly probable (at least if eternal inflation can occur) when one uses the anthropic selection effect of weighting by observers.

A problem with the no-bang proposal is that for it to work, one needs to invoke appropriate quantum fluctuations of the inflaton to produce eternal inflation and yet the asymptotically de Sitter regions produced by the much larger number of non-inflationary states do not produce far more disordered observers or Boltzmann

brains [76, 11, 77, 78, 41, 79, 80, 81, 82, 83, 84, 85, 86, 87, 55, 88] than the much smaller number of inflationary states produce of ordered ordinary observers. That is, the no-bang proposal seems to suffer from the same qualitative problem as the no-boundary proposal in very heavily weighting non-inflationary solutions over inflationary solutions [41], though for the no-bang proposal the quantitative problem is the factor of ‘only’ $e_{\text{no-bang}}^S \sim \exp(4 \times 10^{90})$ rather than the much bigger factor of $e^{S_{\text{as}}} \sim \exp(1.4 \times 10^{122})$ for the no-boundary proposal.

4 Anisotropic necks

So far we have focused on seed states on necks that are nearly homogeneous and isotropic. However, Giddings and Marolf [55] note that there may be far more nonperturbative states, of the order of $e^{S_{\text{as}}} = \exp(\pi b^2/G) \sim \exp(1.4 \times 10^{122})$, where for the numerical values I am using [41] the data of [46, 59]. They cite the conjecture [89, 90] “that the full space of asymptotically (past and future) de Sitter states is of finite dimension, with entropy given by the Bekenstein-Hawking value” and say “we should include black holes (and possibly other non-perturbative gravitational states) up to the maximum size $\sim R$.” Here I find that if one just requires nonsingular seed states on a neck, the number of states can be infinite, but if one requires that these states evolve to just a single asymptotically de Sitter space (in both the future and in the past), the number of states may be finite and of order $e^{S_{\text{no-bang}}}$ rather than of the order of the enormously larger $e^{S_{\text{as}}}$.

A gravitational state that is a highly nonperturbative deviation from de Sitter space is the Kottler [91] or Schwarzschild-de Sitter metric

$$ds^2 = - \left(1 - \frac{2M}{r} - \frac{r^2}{b^2}\right) dt^2 + \left(1 - \frac{2M}{r} - \frac{r^2}{b^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (10)$$

which has both a cosmological constant $\Lambda = 3/b^2$ and a mass parameter M (but no other matter). It has the topology of $R^1 \times R^1 \times S^2$ or $R^1 \times S^1 \times S^2$ if identified to make it spatially compact, with homogeneous isotropic 2-spheres. A metric of this general topology and symmetry can be written as

$$ds^2 = -dt^2 + L^2(t, x)dx^2 + r^2(t, x)(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (11)$$

where x has either an infinite range (the $R^1 \times R^1 \times S^2$ case) or is periodically identified (the $R^1 \times S^1 \times S^2$).

If such a metric has a neck, in the appropriate coordinate system it would be a surface of constant t (say $t = 0$) at which $\int r^2 L dx$ has a minimum. One can always make a local rescaling of x so that $L(0, x) = 1$ at the neck. Then the 3-metric of the neck may be written as

$$ds_{\text{neck}}^2 = dx^2 + r^2(x)(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (12)$$

described by the single function $r(x)$ (and by the periodicity of x in the case in which the neck topology is $S^1 \times S^2$ rather than $R^1 \times S^2$).

In the generic Kottler or Schwarzschild-de Sitter metric, the radius r of the 2-sphere oscillates with the R^1 or S^1 coordinate x , taking values between the two positive roots of $1 - 2M/r + r^2/b^2$. The limiting case in which the two positive roots coalesce (which does *not* make the event horizons at the two roots coalesce, since the proper distance between them remains finite, but only makes their areas the same), $M = 1/\sqrt{9\Lambda} = b/\sqrt{27}$, leads to $r = 1/\sqrt{\Lambda} = b/\sqrt{3}$ being constant over the entire spacetime, and the resulting Nariai metric [92] may be written in the form (11) as

$$ds^2 = -dt^2 + \cosh(\sqrt{3}t/b)dx^2 + (b^2/3)(d\theta^2 + \sin^2\theta d\varphi^2). \quad (13)$$

Unlike the generic Kottler or Schwarzschild-de Sitter, which extends to $r = 0$ and has a curvature singularity there, the Nariai metric, which has r constant everywhere, is completely nonsingular. However, it has no matter and hence no matter entropy. Therefore, let us add thermal radiation to the neck to see how large we can make the matter entropy. Again the condition that the neck is a hypersurface of minimal 3-volume implies that the temperature T of the thermal radiation must be less than the value given by Eq. (4) and can attain that value in the case $\dot{\phi} = 0$. For simplicity, let us assume that the neck is a moment of time symmetry, a hypersurface of zero extrinsic curvature with $\dot{\phi} = 0$, and not just a hypersurface with the trace of the extrinsic curvature is zero.

The Einstein constraint equation for the neck with metric (12) then gives

$$\frac{1 - r'^2 - 2rr''}{r^2} = -G_0^0 = \Lambda + 4\pi G(m^2\phi^2 + 2aT^4), \quad (14)$$

where a prime denotes a derivative with respect to the R^1 or S^1 coordinate x . Since the right hand side of this constraint equation is positive, this equation can certainly be solved for $r(x)$.

In particular, when the right hand side is constant (as a function of x), then there is the homogeneous solution $r = [\Lambda + 4\pi G(m^2\phi^2 + 2aT^4)]^{-1/2}$. If x has length X , then the total 3-volume of this homogeneous neck is $4\pi r^2 X$, and the entropy of the thermal radiation is $S = (16\pi/3)aT^3 r^2 X$. But since the length X can be made arbitrarily large, the entropy of the neck is unbounded above.

That is, a positive cosmological constant does *not* prevent the matter entropy, even on a regular minimal hypersurface, from being infinitely large, because the minimal hypersurface can be chosen to be infinitely large.

This homogeneous example does not give an asymptotically de Sitter space, since its time evolution causes r to collapse to zero at a big bang (in the past) and at a big crunch (in the future). One can see this for the metric (11), because of the Einstein equation

$$-\frac{\ddot{r}}{r} - \frac{r''}{rL^2} + \frac{\dot{r}\dot{L}}{rL} + \frac{r'L'}{rL^3} = \frac{1}{2}(-G_0^0 + G_1^1) = 4\pi G\dot{\phi}^2 + (16\pi G/3)aT^4, \quad (15)$$

assuming for simplicity on the far right hand side that ϕ is independent of x , θ , and φ for each t . The homogeneous initial data gives $r' = 0$ and $L' = 0$, so

$$-\frac{L}{r} \left(\frac{\dot{r}}{L} \right)' = 4\pi G\dot{\phi}^2 + (16\pi G/3)aT^4 > 0, \quad (16)$$

which causes r to collapse to zero size in a finite time.

On the other hand, we can choose inhomogeneous initial data in which $r(x)$ oscillates (and the density of the thermal radiation also oscillates appropriately to solve the constraint equation). In the regions where r'' is sufficiently negative (positive r concave downward as a function of x), one can have $\ddot{r} > 0$, and such regions can expand indefinitely to become asymptotically de Sitter regions in both the infinite past and in the infinite future. Again by having X , the range of x , large enough, one can get an arbitrarily large amount of matter entropy on the neck. Therefore, even if one requires asymptotically de Sitter regions in both the past and the future, a regular neck can have an infinite matter entropy.

One can also do this even if the topology of the neck is S^3 rather than $R^1 \times S^2$ or $S^1 \times S^2$, simply by choosing $r(x)$ on the neck to go to zero (with $r' = \pm 1$ to avoid singularities) at the two ends of the range for x . One can still have an unbounded range of x in between, and hence an unbounded amount of matter entropy on the regular S^3 neck.

Since the regions which expand into asymptotically de Sitter space have $\ddot{r} > 0$, they must have r'' sufficiently negative to solve the Einstein equation (15). This gives a limit on the range of x which evolves to a single asymptotically de Sitter region, and I would conjecture that the matter entropy on that part of the neck is bounded by $S_{\text{no-bang}}$ given by Eq. (7).

If one chooses an oscillating $r(x)$ with a greater range, one will get a nested sequence of asymptotic de Sitter regions with black hole singularities developing in between, rather than a single asymptotic de Sitter region. In particular, at points on the neck where $r' = 0$ and $r'' > 0$, one will have $\ddot{r} < 0$, so at this value of x for both $t < 0$ and for $t > 0$ the 2-spheres of constant (t, x) will be closed trapped surfaces. (Even if the resulting black holes evaporate away by Hawking radiation, it does not seem that this would eliminate the existence of more than one asymptotic de Sitter region in the case in which one has the range of x too large at the neck.)

Therefore, if one limits the quantum states to give a regular neck that evolves into only one asymptotically de Sitter region in both the infinite past and the infinite future, as I have proposed for the no-bang mixed quantum state of the universe, and if my conjecture above is true, then the entropy of this state may indeed be given approximately by $S_{\text{no-bang}}$.

5 Conclusions

In this paper I have proposed the no-bang mixed quantum state for the entire universe or multiverse, which for each positive value of the cosmological constant is an equal mixture of all quantum states that may be generated by group averaging from pure seed states that have a regular minimal hypersurface and which evolve to single asymptotically de Sitter regions in the infinite past and future, without any big bang or big crunch. The total number of these pure states that go into the no-bang mixed state is of the order of the exponential of the three-quarter power of the Bekenstein-Hawking entropy of de Sitter [in $D = 4$ dimensions, otherwise

the exponent is not $3/4$ but $(D - 1)/D]$, at least if most of these pure states are approximately homogeneous and isotropic on the largest scales. This number is enormously less than the number of quantum states normally attributed to de Sitter. The vast majority of even these restricted no-bang states do not have rapid inflation (from an assumed inflaton that can lead to exponential expansion much faster than the asymptotic de Sitter regions). However, if one includes an anthropic selection effect by weighting by the 3-volume at some epoch after possible eternal inflation, then a much smaller number of states may dominate (much less than the apparent possible number of human genomes) and make the resulting conditional probability of rapid inflation very large.

In this way an anthropic analysis of the no-bang quantum state leads to inflation and the possibility of fitting our observations of the universe (given a suitable inflaton, cosmological constant, Standard Model of particle physics, etc., in at least some part of the string or other landscape that is not atypical when weighted by observations). Of course, the no-bang state is just a proposal that is not derived from deeper principles and admittedly at present appears rather *ad hoc*, so it would be desirable to be able to derive it (or an improvement) from simpler or more fundamental principles.

Acknowledgments

I appreciated the hospitality of Edgar Gunzig and the Cosmology and General Relativity symposium of the Peyresq Foyer d'Humanisme in Peyresq, France, where Donald Marolf kindly and patiently explained to me many aspects of quantum de Sitter space. Andrei Linde and Tom Banks have provided helpful comments on an earlier version of the manuscript. This research was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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